Dungog High School

Higher School Certificate Courses

Mathematics Faculty

2012

Assessment Booklet
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Information for Students

- This booklet is an important document. It explains the outcomes of the course you are doing and how your assessment mark will be determined. It also gives a breakdown of all the content that is involved in each course allowing students to self check they have covered all the work. The performance bands included, explain what you need to do to demonstrate your knowledge, understanding and skills.

- One assessment task may be used to measure a variety of outcomes.

- If you miss lessons it is your responsibility to find out if any information about assessment tasks were given out during the period of absence in addition to catching up any missed work. In cases of prolonged absence you should request that school work be sent home for you to complete.

- If you don’t understand what is required of you in any assessment task it is your responsibility to seek help from your class teacher or the Head Teacher.

- You will need to attend each lesson and complete all set work. The Board may refuse to grant a Higher School Certificate to a student whose application at school has been unsatisfactory. This could mean that poor attendance may result in you being deemed as not satisfactorily completing the course.

It is your responsibility to carefully read and understand this information.
GENERAL MATHEMATICS

HSC Board of Studies Outcomes

H1 Appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society.

H2 Integrates mathematical knowledge and skills from different content areas in exploring new situations.

H3 Develops and tests a general mathematical relationship from observed patterns.

H4 Analyses representations of data in order to make inferences, predictions and conclusions.

H5 Makes predictions about the behaviour of situations based on simple models.

H6 Analyses two-dimensional and three-dimensional models to solve practical and mathematical problems.

H7 Interprets the results of measurements and calculations and makes judgements about reasonableness.

H8 Makes informed decisions about financial situations.

H9 Develops and carries out statistical processes to answer questions which she/he and others have posed.

H10 Solves problems involving uncertainty using basic principles of probability.

H11 Uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

HSC Course Content

Financial Mathematics – 30 hours
  FM4: Credit and Borrowing
  FM5: Annuities and loan repayments
  FM6: Depreciation

Data Analysis – 26 hours
  DA5: Interpreting sets of data
  DA6: The normal distribution
  DA7: Correlation

Measurement – 24 hours
  M5: Further applications of area and volume
  M6: Applications of trigonometry
  M7: Spherical geometry

Probability – 16 hours
  PB3: Multi-stage events
  PB4: Applications of probability

Algebraic Modelling – 24 hours
  AM3: Algebraic skills and techniques
  AM4: Modelling linear and non-linear relationships

*Note: Hours shown are indicative only*
Financial Mathematics

**FM4: Credit and borrowing**
This unit of work focuses on the mathematics involved in borrowing money, the different types of loans available and credit cards.

**Students learn and acquire the following skills, knowledge and understanding**
- calculation of principal, interest and repayments for flat-rate loans
- calculation of values in a table of home loan repayments
- comparison of different options for borrowing money in relation to total repayments, fees, interest rates and flexibility
- calculation of credit-card payments, incorporating fees, charges, rates and interest-free periods
- use of published tables from financial institutions to determine monthly repayments on a reducing balance loan.

**FM5: Annuities and loan repayments**
The principal focus of this unit is the nature and mathematics of annuities; the processes by which they accrue and the ways of maximising their value as an investment. Emphasis should be placed on using formulae and tables.

**Students learn and acquire the following skills, knowledge and understanding**
- recognition that an annuity is a form of investment involving periodical, equal contributions to an account, with interest compounding at the conclusion of each period
- calculation of the future value \( A \) of an annuity (or the contribution per period), using
  \[
  A = M \left( \frac{(1+r)^n - 1}{r} \right)
  \]
  where \( M \) = contribution per period, paid at the end of the period.
  Note: the future value of an annuity is the total value of the investment at the conclusion of the last period for payment.
  For example, I am planning to take the trip of a lifetime in ten years’ time and estimate that the amount of money I will need at that time is $30 000. I am advised to contribute $2500 each year into an account that pays 4% pa, compounded annually. Will I have enough money in ten years time to make my dream come true? By how much will I fall short of or overshoot my goal?
- calculation of the present value \( N \) of an annuity (or the contribution per period), using
  \[
  N = M \left( \frac{1}{r} \right) \left( \frac{(1+r)^n - 1}{(1+r)^n} \right)\]
  or
  \[
  N = \frac{A}{(1+r)^n}
  \]
  Note: the present value of an annuity is the single sum of money which, if invested today at the rate of compound interest which applies to the annuity, would produce the same financial result over the same period of time.
  For example, which would give the better financial result at the end of 20 years — a lump sum of $100 000 invested today at 12% pa compounded annually, or a monthly payment of $1 000, commencing today, with interest of 12% pa compounded monthly?
- using tables to solve problems involving annuities
- use the present value formula for annuities to calculate loan instalments, and hence the total amount paid over the term of a loan
- investigate various processes for repayment of loans
- calculate the fees and charges which apply to different options for borrowing money in order to make a purchase.

**FM6: Depreciation**
The focus of this unit is to investigate situations involving the depreciation of an asset over time.

**Students learn and acquire the following skills, knowledge and understanding**
- modelling depreciation by using appropriate graphs, tables and functions
- using formulae for depreciation:
  (a) the straight line method \( S = V_0 - D_n \), where \( S \) = salvage (current) value of asset, \( D \) = amount of depreciation apportioned per period, \( V_0 \) = purchase price of the asset, and \( n \) = total number of periods
  (b) the declining balance method \( S = V_0(1-r)^n \), where \( S \) is the salvage value after \( n \) periods, \( V_0 \) is the purchase price of the asset and \( r \) is the percentage interest rate per period, expressed as a decimal
• preparing tables of values and hence developing graphs of against \( n \) for different values of \( r \)
  Note: these are examples of exponential decay (see AM3, AM4)
• comparing the results obtained through each method
• using the above formulae to create and compare depreciation tables
• calculating tax deductions based on depreciation of assets.

Data Analysis

DA5: Interpreting sets of data
The principal focus of this unit is the use of data displays, measures of location and measures of spread to summarise and interpret one or more sets of data.

Students learn and acquire the following skills, knowledge and understanding
• identifying measures of location as mean and median
• identifying measures of spread as range, interquartile range and standard deviation
• investigating outliers in small data sets and their effects on the mean, median and mode
• describing the general shape of a graph or display which represents a given data set, eg in terms of smoothness, symmetry or number of modes
• making judgements about the data based on observed features of the display such as shape and skewness
• displaying data in double (back-to-back) stem-and-leaf plots
• displaying data in two box-and-whisker plots drawn on the same scale
• displaying two sets of data on a radar chart
• preparing an area chart to illustrate and compare different sets of data over time (see example at end of unit)
• using multiple displays to describe and interpret the relationships between data sets
• interpreting data presented in two-way table form, eg male/female versus exercise/no exercise
• comparing summary statistics from two sets of data.

DA6: The normal distribution
In this unit, students will apply the properties of the standard normal distribution to the solution of real problems.

Students learn and acquire the following skills, knowledge and understanding
• describing the z-score (standardised score) corresponding to a particular score in a set of scores as a number indicating the position of that score relative to the mean
• using the formula \( z = \frac{x - \bar{x}}{s} \) to calculate z-scores, where \( s \) is the standard deviation
  \( (s = \sigma_n \text{ for a population, } s = \sigma_{n-1} \text{ for a sample}) \)
• using calculated z-scores to compare scores from different data sets
• identifying the properties of data that are normally distributed, ie
  – the mean, median and mode are equal
  – if represented by a histogram, the resulting frequency graph is ‘bell shaped’
• using collected data to illustrate that, for normally distributed data:
  – approximately 68% of scores will have z-scores between –1 and 1
  – approximately 95% of scores will have z-scores between –2 and 2
  – approximately 99.7% of scores will have z-scores between –3 and 3
• using these measures to make judgements in individual cases.

DA7: Correlation
In this unit, students investigate the strength of association of data through examining a scatterplot of ordered pairs. Where appropriate, students find the equation of a line of fit and use the equation to make predictions.

Students learn and acquire the following skills, knowledge and understanding
• plotting ordered pairs of data onto a scatterplot
• recognising from the scatterplot:
  – whether the points appear to form a mathematical pattern
  – whether the pattern appears to be linear
• establishing a median regression line to give a line of fit on a scatterplot with a ruler and pencil
• measuring the gradient of the line of fit drawn, with ruler and pencil
• noting the vertical intercept of the line of fit drawn
• establishing the equation of the resulting line of fit in form $y = mx + b$ (see AM2)
• using this equation to make predictions.

The remaining points relate to correlation. Students will not be required to calculate correlation coefficients.

• interpreting the strength of association using a given correlation coefficient
• interpreting the sign of a given correlation coefficient
• recognising that a high degree of correlation does not necessarily imply causality, eg there is a very high correlation between the sizes of one’s left and right feet, but one does not cause the other.

Measurement

**M5: Further applications of area and volume**

In this unit, the work commenced in M 2: Applications of area and volume is extended to include surface area and volume of complex figures, and the use of approximations in calculating area and volume of irregular figures.

**Students learn and acquire the following skills, knowledge and understanding**

• calculating areas of ellipses, annuluses and parts of a circle (quadrant, sector) using appropriate formulae
• calculating areas of composite figures
• applying Simpson’s rule over three equally spaced points, ie one application (problems involving five points should be treated using two applications)
• calculating external surface area of open (without top and/or bottom) and closed cylinders
• calculating surface area of spheres
• calculating volumes of composite solids
• determining errors in calculations resulting from errors in measurement.

**M6: Applications of trigonometry**

This unit extends students’ knowledge of trigonometry and area to include non-right-angled triangles. Problems to be solved will incorporate practical work with offset and radial surveys. Angles will be approximated to the nearest minute.

**Students learn and acquire the following skills, knowledge and understanding**

• solving problems using trigonometric ratios in one or more right-angled triangles
• using compass bearings (eight points only) and true bearings (three-figure bearings) in problem-solving related to maps and charts
• establishing the sine, cosine and tangent ratios for obtuse angles from a calculator
• determining the sign of the above ratios for obtuse angles
• preparing diagrams to represent given information
• using the sine rule to find lengths and angles $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Note: It is not intended that students study the ambiguous case of the sine rule.

• calculating area of a triangle using the formula $A = \frac{1}{2}ab\sin C$

• using the cosine rule to find lengths and angles $c^2 = a^2 + b^2 - 2bc\cos C$ or $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

• using appropriate trigonometric ratios and formulae in two-triangle problems where one triangle is right-angled and the diagram is given
• solving problems involving non-right-angled triangles
• selecting and using appropriate trigonometric ratios and formulae to solve problems
• conducting radial (both plane table and compass) surveys
• solving problems involving non-right-angled triangle trigonometry, Pythagoras’ theorem and area in offset and radial surveys.
M7: Spherical geometry

In this unit, geometry and trigonometry are applied to solve problems relating to the Earth as a sphere. Applications include locating positions on the Earth, using latitude and longitude, and calculating time differences.

**Students learn and acquire the following skills, knowledge and understanding**

- calculating arc lengths of a circle
- distinguishing between great and small circles
- using the Equator and the Greenwich Meridian as lines of reference for locations on the Earth’s surface
- locating positions on the globe using latitude and longitude
- converting nautical miles (M) to kilometres and vice versa, given $1.852 \text{ km} = 1 \text{ M}$
- calculating distances between two points on the same great circle in nautical miles and kilometres (radius of the Earth to be taken as 6400 km)
- defining 1 knot as a speed of 1 M per hour
- using time zones and the International Date Line in solving problems
- calculating time differences given the difference in longitudes (Apply $15^\circ = 1$ hour and $1^\circ = 4$ minutes time difference. Daylight-saving time is to be considered.)
- determining times for cities in different countries in related travel questions.

Probability

PB3: Multi-stage events

The focus of this unit is on counting the number of outcomes for an experiment, or the number of ways in which an event may occur. The probability of particular outcomes may then be established. The formulae using factorial notation are not required.

**Students learn and acquire the following skills, knowledge and understanding**

- constructing and using a tree diagram to establish the sample space for a simple multistage event
- multiplying the number of choices at each stage to determine the number of outcomes for a multi-stage event
- establishing that the number of ways in which $n$ different items can be arranged is $n(n-1)(n-2) \times 1$ eg the number of arrangements of 4 different items is $4 \times 3 \times 2 \times 1 = 24$; the number of arrangements of 3 different items is $3 \times 2 \times 1 = 6$
- checking that these results are true by listing arrangements for small numbers of items
- establishing the number of ordered selections that can be made from a group of different items (small numbers only), eg if selecting two particular positions (such as captain and vice-captain) from a team of five people, the number of selections is $5 \times 4 = 20$
- verifying these results by listing ordered selections (small numbers only)
- establishing the number of unordered selections that can be made from a group of different items (small numbers only), eg if selecting a pair of people to represent a team of five, the number of selections is half of the number of ordered selections
- verifying by listing unordered selections (small numbers only)
- using the formula for the probability of an event to calculate the probability that a particular selection will occur
- using probability tree diagrams to solve problems involving two-stage events.

PB4: Applications of probability

In this unit, students calculate expected outcomes from simple experiments and compare them with experimental results.

**Students learn and acquire the following skills, knowledge and understanding**

- calculating the expected number of times a particular outcome would arise, given the number of trials of a simple experiment, by establishing the theoretical probability and multiplying by the number of trials
- comparing the above result with an experimental result
- calculating financial expectation by multiplying each financial outcome by its probability and adding the results together
  Note: A financial loss is regarded as negative.
- carrying out simulations to model events, eg tossing a coin with the outcomes representing the sex of the offspring
drawing up a table (two-way table) to illustrate results gained on a test designed to determine the existence (in a particular case) of a phenomenon which has a low overall probability of occurrence, eg screening for medical conditions, or using a lie detector to indicate guilt or innocence

interpreting the information in the table and making judgements about the conclusions established by the test.

Algebraic Modelling

**AM3: Algebraic skills and techniques**
This unit develops algebraic skills and techniques that are used in work-related and everyday contexts. As far as possible, real contexts should be used to demonstrate the use of algebra in practical life.

**Students learn and acquire the following skills, knowledge and understanding**

- substituting into and evaluating algebraic expressions — linear, quadratic, cubic, as well as those involving square and cube roots, such as $r = \frac{\sqrt[3]{3V}}{4\pi}$
- adding and subtracting like terms
- multiplying and dividing algebraic terms and expressions
- changing the subject of equations and formulae involving linear and quadratic terms, eg make $s$ the subject of $v^2 = u^2 + 2as$
- solving equations after substituting values, eg evaluate $t$ when $d = 5t^2$ and $d = 300$
- solution of equations arising from practical situations by estimation and refinement, eg if $x$ is the number of years required for an investment to double at 5% pa compound interest, then $1.05^x = 2$. Find the value of $x$ by making an initial estimate and refining the result, using a calculator
- using positive and negative powers of ten as part of expressing measurements in scientific notation.

**AM4: Modelling linear and non-linear relationships**
This unit focuses on the examination of practical problems that can be modelled algebraically.

**Students learn and acquire the following skills, knowledge and understanding**

- generating tables of values and graphing linear functions with pencil and paper
- interpretation of the point of intersection of the graphs of two linear functions drawn from practical contexts, eg ‘breakeven’ points (see applications)
- generating tables of values and graphing quadratic functions of the form $y = ax^2 + bx + c$, $x \geq 0$ with pencil and paper
- noting that different forms of an expression produce identical graphs, eg $y = (x - 2)^2 + 3$, $y = x^2 - 4x + 7$
- using a quadratic graph to find maximum and minimum values in practical contexts
- generating tables of values and graphing cubic, exponential and hyperbolic functions with pencil and paper. Functions to be restricted to the following forms — cubic: $y = ax^3$, $x \geq 0$; exponential: $y = b(a^x)$, $x \geq 0$; hyperbolic $y = \frac{a}{x}$, $x > 0$
- recognition that, for $a > 1$, $y = b(a^x)$ represents exponential growth and, for $0 < a < 1$, it represents exponential decay (see FM2, FM6)
- development of equations such as $y = ax^2$, $h = at^3$ from descriptions of situations in which one quantity varies directly as the power of another
- development of an equation such as $y = \frac{a}{x}$ from a description of a situation in which one quantity varies inversely with another
- subsequent evaluation of $a$ in the equations shown in the above two points, given one pair of variables, and using the resulting formula to find other values of the variables
- using algebraic functions as models of physical phenomena
- recognising the limitations of models when interpolating and/or extrapolating.
# Dungog High School Mathematics Faculty
## Teaching Program Timeline
### Year 12 General Mathematics Course 2011-2012

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**Week 1**
- Credit & Borrowing (FM4)

**Week 2**
- Interpreting Data (DA5)

**Week 3**
- Area & Volume (M5)

**Week 5** (No Students)

#### Term 1 12

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**Week 2**
- Annuities & Loans (FM5)

**Week 3**
- Normal Distribution (DA6)

**Week 4**
- Trigonometry (M6)

**Week 6**
- Algebraic Skills & Techniques (AM3)

#### Term 2 12

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**Week 1**
- Half Yearly (Test 5)

**Week 2**
- Multi-Stage Events & Probability (PB3 & PB4)

**Week 4**
- Modelling Linear & Non-Linear Relationships (AM4)

**Week 9**
- Spherical Geometry (M7)

#### Term 3 12

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**Week 1**
- Depreciation (FM6) & Correlation (DA7)

**Week 8**
- Trial HSC (Test 8)

**Week 10**
- Revision

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**Note:** The position of the tests on this timeline, are only suggestive. Actual written notification will be given to every student at least 2 weeks prior to the assessment task.
# HSC Assessment Grid

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## Nature of Assessment Task

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<th>Written Test (%)</th>
<th>Trial HSC (%)</th>
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<tbody>
<tr>
<td>Knowledge &amp; Skills</td>
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<tr>
<td>Application</td>
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<tr>
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Note: The dates of tasks are flexible.

This table should be read in conjunction with the above table and the General Mathematics Timeline.

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<th>Task</th>
<th>Test</th>
<th>Topics</th>
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<tbody>
<tr>
<td>1</td>
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<td></td>
<td>T2</td>
<td>Interpreting Data</td>
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<td></td>
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<td>Area &amp; Volume</td>
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<td>T3</td>
<td>Annuities &amp; Loan Repayments</td>
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<td>The Normal Distribution</td>
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<td>Algebraic Skills and Techniques</td>
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<td>Recall of all previous topics</td>
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<td>Multistage Events &amp; Probability</td>
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<td>Modelling Linear and Non-linear Relationships</td>
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</table>

-11-
**MATHEMATICS**

**HSC Board of Studies Outcomes**

H1  Seeks to apply mathematical techniques to problems in a wide range of practical contexts.

H2  Constructs arguments to prove and justify results.

H3  Manipulates algebraic expressions involving logarithmic and exponential functions.

H4  Expresses practical problems in mathematical terms based on simple given models.

H5  Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6  Uses the derivative to determine the features of the graph of a function.

H7  Uses the features of a graph to deduce information about the derivative.

H8  Uses techniques of integration to calculate areas and volumes.

H9  Communicates using mathematical language, notation, diagrams and graphs.

**HSC Course Content**

There are 8 topics covered in this course:

- Coordinate methods in geometry
- Geometrical applications of differentiation
- Integration
- Trigonometric functions (including applications of trigonometric ratios)
- Logarithmic and exponential functions
- Applications of calculus to the physical world
- Probability
- Series and series applications

*See Extension 1 Mathematics Course Content for break down of topics.*
<table>
<thead>
<tr>
<th>Term 4 2011</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Week 2</td>
<td>Week 3</td>
</tr>
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<td>Geometrical Applications of Differentiation</td>
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<table>
<thead>
<tr>
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<th>Test 3</th>
<th>Test 4</th>
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<tbody>
<tr>
<td>Week 1</td>
<td>Week 2</td>
<td>Week 3</td>
</tr>
<tr>
<td>(No Students)</td>
<td>Trigonometric Functions</td>
<td>Probability</td>
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<table>
<thead>
<tr>
<th>Term 2 2012</th>
<th>Test 6</th>
<th>Test 7</th>
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<tr>
<td>Week 1</td>
<td>Week 2</td>
<td>Week 3</td>
</tr>
<tr>
<td>Half Yearly (Test 5)</td>
<td>Applications of Calculus to the Physical World</td>
<td>Series and Applications</td>
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<table>
<thead>
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<tbody>
<tr>
<td>Week 1</td>
<td>Week 2</td>
</tr>
<tr>
<td>Co-ordinate Methods in Geometry (Revisited)</td>
<td>Trial HSC (Test 8)</td>
</tr>
</tbody>
</table>

*Note: The position of the tests on this timeline, are only suggestive. Actual written notification will be given to every student at least 2 weeks prior to the assessment task.*
## HSC ASSESSMENT GRID
### SUBJECT: MATHEMATICS

<table>
<thead>
<tr>
<th>Syllabus Components</th>
<th>Syllabus Weighting (%)</th>
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### Nature of Assessment Task

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<th>Written Test (%)</th>
<th>Half Yearly (%)</th>
<th>Written Test (%)</th>
<th>Trial HSC (%)</th>
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<td>22</td>
<td>18</td>
</tr>
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</table>

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EXTENSION 1 MATHEMATICS

HSC Board of Studies Outcomes

HE1 Appreciates interrelationships between ideas drawn from different areas of mathematics.

HE2 Uses inductive reasoning in the construction of proofs.

HE3 Uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay.

HE4 Uses the relationship between functions, inverse functions and their derivatives.

HE5 Applies the chain rule to problems including those involving velocity and acceleration as functions of displacement.

HE6 Determines integrals by reduction to a standard form through a given substitution.

HE7 Evaluates mathematical solutions to problems and communicates them in an appropriate form.

HSC Course Content

There are 11 topics covered in this course:

- Methods of integration including Primitive of sin²x and cos²x
- Rates of Change
- Velocity & acceleration as a function of x
- Projectile motion
- Simple harmonic motion
- Inverse functions and inverse trigonometric functions
- Induction
- Binomial theorem
- Further probability
- Iterative methods for numerical estimation of the roots of a polynomial equation
- Harder applications of HSC Mathematics questions

Explanation of symbols

†: denotes that students are not required to reproduce proofs of results contained in items preceded by this symbol.

E: denotes that the following item or items are not included in the mathematics course but are in the extension course.

Geometrical Applications of Differentiation

- Significance of the sign of the derivative.
- Stationary points on curves.
- The second derivative. The notations \( f''(x) \), \( \frac{d^2y}{dx^2} \), \( y'' \).
- Geometrical significance of the second derivative.
- The sketching of simple curves.
- Problems on maxima and minima.
- Tangents and normals to curves.
- The primitive function and its geometrical interpretation.

Binomial Theorem

- (E) Expansion of \((1 + x)^n\) for \(n = 2,3,4,\ldots\) Pascal Triangle. Proof of the Pascal Triangle relations. Extension to the expansion \((a + x)^n\).
- (E,†) Proof by Mathematical Induction of the formula for \(^nC_k\) (also denoted by \( \binom{n}{k} \)).
• (E) Finite series and further properties of binomial coefficients.

Integration
• (†) The definite integral.
• (†) The relation between the integral and the primitive function.
• (†) Approximate methods: trapezoidal rule and Simpson’s rule.
• Applications of integration: areas and volumes of solids of revolution.
• (E) Methods of integration, including reduction to standard forms by very simple substitutions.

Polynomials
• (E) Iterative methods for numerical estimation of the roots of a polynomial equation.

The Trigonometric Functions
• Circular measure of angles. Angle, arc, sector.
• The functions sin x, cos x, tan x, cosec x, sec x, cot x and their graphs.
• Periodicity and other simple properties of the functions sin x, cos x and tan x.
• Approximations to sin x, cos x, tan x, when x is small. The result \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \).
• (†) Differentiation of cos x, sin x, tan x.
• Primitive functions of sin x, cos x, sec^2 x.
• (E) Primitive functions of sin^2 x and cos^2 x.
• Extension of points 2 to 6 to functions of the form a sin(bx + c), etc.

Inverse Functions and the Inverse Trigonometric Functions
• (E) Discussion of inverse function. The functions \( y = \log_a x \) and \( y = a^x \) as inverse functions. The relation \( \frac{dy}{dx} \cdot \frac{dx}{dy} = 1 \).
• (E) The inverse trigonometric functions.
• (E) The graphs of sin^{-1}x, cos^{-1}x, tan^{-1}x.
• (E) Simple properties of the inverse trigonometric functions.
• (E) The derivatives of sin^{-1}(x/a), cos^{-1}(x/a), tan^{-1}(x/a), and the corresponding integrations.

Probability
• Random experiments, equally likely outcomes; probability of a given result.
• Sum and product of results.
• Experiments involving successive outcomes; tree diagrams.

Logarithmic and Exponential Functions
• Review of index laws, and definition of \( ar \) for \( a > 0 \), where \( r \) is rational.
• (†) Definition of logarithm to the base \( a \). Algebraic properties of logarithms and exponents.
• (†) The functions \( y = a^x \) and \( y = \log_a x \) for \( a > 0 \) and real x. Change of base.
• (†) The derivatives of \( y = a^x \) and \( y = \log_a x \). Natural logarithms and exponential function.
• Differentiation and integration of simple composite functions involving exponentials and logarithms.

Applications of Calculus to the Physical World
• Rates of change as derivatives with respect to time. The notation \( \dot{x}, \ddot{x} \), etc.
• (†) Exponential growth and decay; rate of change of population; the equation \( \frac{dN}{dt} = kN \), where \( k \) is the population growth constant.
• (E) The equation \( = k (N - P) \), where \( k \) is the population growth constant, and \( P \) is a population constant.
• Velocity and acceleration as time derivatives. Applications involving:
  - the determination of the velocity and acceleration of a particle given its distance from a point as a function of time;
- the determination of the distance of a particle from a given point, given its acceleration or velocity as a function of time together with appropriate initial conditions.
- (E) Velocity and acceleration as functions of $x$.
- (E) Applications in one and two dimensions (projectiles).
- (E) Description of simple harmonic motion from the equation $x = a \cos (nt + \phi)$, $a > 0$, $n > 0$. The differential equation of the motion.

**Series and Applications**
- Arithmetic series. Formulae for the $n$th term and sum of $n$ terms.
- Geometric series. Formulae for the $n$th term and sum of $n$ terms.
- Geometric series with a ratio between $-1$ and $1$. The limit of $x^n$, as $n \to \infty$, for $|x| < 1$, and the concept of limiting sum for a geometric series.
- (E) Mathematical induction. Applications.
- Applications of arithmetic series. Applications of geometric series: compound interest, simplified hire purchase and repayment problems. Applications to recurring decimals.
# Teaching Program Timeline

## Year 12 Ext 1 Mathematics Course 2011-2012

### Term 4 11

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
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<th>Week 10</th>
<th>Week 11</th>
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<td>Approximate Roots of Polynomials</td>
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### Term 1 12

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<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
<th>Week 11</th>
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<tbody>
<tr>
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<td>Inverse Functions and Inverse Trigonometry</td>
<td>Further Integration</td>
<td>Further Probability</td>
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### Term 2 12

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<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
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<tbody>
<tr>
<td>Half Yearly (Test 5)</td>
<td>Rates of Change</td>
<td>Velocity and Acceleration as a function of x</td>
<td>Simple Harmonic Motion</td>
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### Term 3 12

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<th>Week 7</th>
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<th>Week 9</th>
<th>Week 10</th>
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<tbody>
<tr>
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<td>Trial HSC (Test 8)</td>
<td>Revision</td>
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# HSC Assessment Grid
## Subject: Mathematics Extension 1

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<tr>
<th>Syllabus Components</th>
<th>Syllabus Weighting (%)</th>
<th>Task 1</th>
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<th>Task 3</th>
<th>Task 4</th>
<th>Task 5</th>
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<tbody>
<tr>
<td>Knowledge &amp; Skills</td>
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<td>7</td>
<td>8</td>
<td>6</td>
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</tr>
<tr>
<td>Application</td>
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<td>11</td>
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**Nature of Assessment Task**

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<tr>
<th></th>
<th>Written Test (%)</th>
<th>Written Test (%)</th>
<th>Half Yearly (%)</th>
<th>Written Test (%)</th>
<th>Trial HSC (%)</th>
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<tbody>
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Note: The dates of tasks are flexible.

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<th>Topics</th>
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<tr>
<td></td>
<td>T2</td>
<td>Approximating Roots of Polynomials</td>
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<tr>
<td></td>
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<td>Induction</td>
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<td>2</td>
<td>T4</td>
<td>Further Integration</td>
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<td>Further Probability</td>
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<td>T5</td>
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<td>T7</td>
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<td>Projectiles</td>
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<tr>
<td></td>
<td></td>
<td>(Trial HSC)</td>
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</tbody>
</table>
EXTENSION 2 MATHEMATICS

HSC Board of Studies Outcomes

E1 Appreciates the creativity, power and usefulness of mathematics to solve a broad range of problems.

E2 Chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings.

E3 Uses the relationship between algebraic and geometric representations of complex numbers and of conic sections.

E4 Uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials.

E5 Uses ideas and techniques from calculus to solve problems in mechanics involving resolution of forces, resisted motion and circular motion.

E6 Combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions.

E7 Uses the techniques of slicing and cylindrical shells to determine volumes.

E8 Applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems.

E9 Communicates abstract ideas and relationships using appropriate notation and logical argument.

HSC Course Content

There are 8 topics covered in this course:
- Graphs
- Complex Numbers
- Conics
- Integration
- Volumes
- Mechanics
- Polynomials
- Harder Extension 1 topics

Graphs

Basic Curves
- graph a linear function \((ax + by + c = 0, y = mx + b)\)
- graph a quadratic function \((y = ax^2 + bx + c)\)
- graph a cubic function \((y = ax^3 + bx^2 + cx + d)\)
- graph a quartic function \((y = ax^4 + bx^3 + cx^2 + dx + e)\)
- graph a rectangular hyperbola \((xy = k)\)
- graph a circle \((x^2 + y^2 + 2gx + 2fy + c = 0)\)
- graph an exponential function \((y = a^x \text{ for both cases } a > 1 \text{ and } 0 < a < 1)\)
- graph a logarithmic function \((y = \log_a x)\)
- graph trigonometric functions \((\text{eg } y = k + a \sin(bx + c))\)
- graph inverse trigonometric functions \((\text{eg } y = a \sin^{-1} bx)\)
- graph the functions \(y = x^{\frac{1}{2}}\) and \(y = x^{\frac{1}{3}}\).

Drawing graphs by addition and subtraction of ordinates
- graph a function \(y = f(x) \pm c\) by initially graphing \(y = f(x)\)
- graph a function \(y = f(x) \pm g(x)\) by initially graphing \(y = f(x)\) and \(y = g(x)\).
Drawing graphs by reflecting functions in coordinate axes
• graph \( y = -f(x) \) by initially graphing \( y = f(x) \)
• graph \( y = |f(x)| \) from the graph of \( y = f(x) \)
• graph \( y = f(-x) \) by initially graphing \( y = f(x) \).

Sketching functions by multiplication of ordinates
• graph a function \( y = cf(x) \) by initially graphing \( y = f(x) \)
• graph a function \( y = f(x) \cdot g(x) \) by initially graphing \( y = f(x) \) and \( y = g(x) \).

Sketching functions by division of ordinates
• graph a function \( y = 1/f(x) \) by initially graphing \( y = f(x) \)
• graph a function \( y = f(x)/g(x) \) by initially graphing \( y = f(x) \) and \( y = g(x) \).

Drawing graphs of the form \([f(x)]^n\)
• graph a function \( y = [f(x)]^n \) by first graphing \( y = f(x) \).

Drawing graphs of the form \( \sqrt{f(x)} \)
• graph a function \( y = \sqrt{f(x)} \) by first graphing \( y = f(x) \).

General approach to curve sketching
• use implicit differentiation to compute \( \frac{dy}{dx} \) for curves given in implicit form
• use the most appropriate method to graph a given function or curve.

Using graphs
• solve an inequality by sketching an appropriate graph
• find the number of solutions of an equation by graphical considerations
• solve problems using graphs.

Complex Numbers
Arithmetic of complex numbers and solving quadratic equations
• appreciate the necessity of introducing the symbol \( i \), where \( i^2 = -1 \), in order to solve quadratic equations
• write down the real part \( \text{Re}(z) \) and the imaginary part \( \text{Im}(z) \) of a complex number \( z = x + iy \)
• add, subtract and multiply complex numbers written in the form \( x + iy \)
• find the complex conjugate \( \bar{z} \) of the number \( z = x + iy \)
• divide a complex number \( a + ib \) by a complex number \( c + id \)
• write down the condition for \( a + ib \) and \( c + id \) to be equal
• prove that there are always two square roots of a non-zero complex number
• find the square roots of a complex number \( a + ib \)
• solve quadratic equations of the form \( ax^2 + bx + c = 0 \), where \( a, b, c \) are complex.

Geometric representation of a complex number as a point
• appreciate that there exists a one to one correspondence between the complex number \( a + ib \) and the ordered pair \((a,b)\)
• plot the point corresponding to \( a + ib \) on an Argand diagram
• define the modulus \( |z| \) and argument \( \arg(z) \) of a complex number \( z \)
• find the modulus and argument of a complex number
• write \( a + ib \) in modulus-argument form
• prove basic relations involving modulus and argument
• use modulus-argument relations to do calculations involving complex numbers.
• recognise the geometrical relationships between the point representing \( z \) and points representing \( \bar{z} \), \( cz \) (\( c \) real) and \( iz \).
Geometrical representations of a complex number as a vector
- appreciate that a complex number $z$ can be represented as a vector on an Argand diagram
- appreciate the geometrical significance of the addition of two complex numbers
- given the points representing $z_1$ and $z_2$, find the position of the point representing $z$, where $z = z_1 + z_2$
- appreciate that the vector representing $z = z_1 + z_2$ corresponds to the diagonal of a parallelogram with vectors representing $z_1$ and $z_2$ as adjacent sides
- given vectors $z_1$ and $z_2$, construct vectors $z_1 - z_2$ and $z_2 - z_1$
- given $z_1$ and $z_2$, construct the vector $z_1 z_2$
- prove geometrically that $|z_1 + z_2| \leq |z_1| + |z_2|

Powers and roots of complex numbers
- prove, by induction, that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ for positive integers $n$
- prove that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ for negative integers $n$
- find any integer power of a given complex number
- find the complex $n^{th}$ roots of $\pm 1$ in modulus-argument form
- sketch the $n^{th}$ roots of $\pm 1$ on an Argand diagram
- illustrate the geometrical relationship connecting the $n^{th}$ roots of $\pm 1$.

Curves and Regions
- given equations $\text{Re}(z) = c, \text{Im}(z) = k (c, k \text{ real})$, sketch lines parallel to the appropriate axis
- given an equation $|z - z_1| = |z - z_2|$ sketch the corresponding line
- given equations $|z| = R, |z - z_1| = R$, sketch the corresponding circles
- given equations $\arg z = \theta, \arg(z - z_1) = \theta$, sketch the corresponding rays
- sketch regions associated with any of the above curves (e.g. the region corresponding to those $z$ satisfying the inequality $|z - z_1| \leq R$)
- give a geometrical description of any such curves or regions
- sketch and describe geometrically the intersection and/or union of such regions
- sketch and give a geometrical description of other simple curves and regions.

Conics
The Ellipse
- write down the defining equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of an ellipse with centre the origin
- sketch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, showing points of intersection with axes of symmetry
- find the lengths of the major and minor axes and semi-major and semi-minor axes of an ellipse
- write down the parametric coordinates $(a \cos \theta, b \sin \theta)$ of a point on an ellipse
- sketch an ellipse using its auxiliary circle
- find the equation of an ellipse from its focus-directrix definition
- find the eccentricity from the defining equation of an ellipse
- given the equation of an ellipse, find the coordinates of the foci and equations of the directrices
- sketch an ellipse, marking on the sketch the positions of its foci and directrices
- use implicit differentiation to find the equations of the tangent and the normal at $P(x_1, y_1)$ on an ellipse
- find the equations of the tangent and the normal at $P(a \cos \theta, b \sin \theta)$ on an ellipse
- find the equation of a chord of an ellipse
- find the equation of a chord of contact
- prove that the sum of the focal lengths is constant
- prove the reflection property, namely that the tangent to an ellipse at a point $P$ on it is equally inclined to the focal chords through $P$
- prove that the chord of contact from a point on a directrix is a focal chord.
- prove that the part of the tangent between the point of contact and the directrix subtends a right angle at the corresponding focus.
prove simple properties for both general ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and for ellipses with given values of \( a \) & \( b \).

The Hyperbola

- write down the defining equation \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) of a hyperbola with centre the origin
- sketch the hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \), showing points of intersection with axes of symmetry and positions of asymptotes
- find the length of major and minor axes and semi-major and semi-minor axes of a hyperbola
- write down the parametric coordinates \((a \sec \theta, b \tan \theta)\) of a point on the hyperbola
- find the equation of a hyperbola from its focus-directrix definition
- find the eccentricity from the defining equation of a hyperbola
- given the equation of the hyperbola, find the coordinates of its foci and equations of its directrices
- sketch a hyperbola, marking on the positions of its foci and directrices
- use implicit differentiation to find the equations of the tangent and the normal at \( P(x_1, y_1) \) on a hyperbola
- find the equations of the tangent and the normal at \( P(a \sec \theta, b \tan \theta) \) on the hyperbola
- find the equation of a chord of a hyperbola
- find the equation of a chord of contact
- prove that the difference of the focal lengths is constant
- prove the reflection property for a hyperbola
- prove that the chord of contact from a point on the directrix is a focal chord
- prove simple properties for both the general hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) and for hyperbolae with given values of \( a \) and \( b \).

The Rectangular Hyperbola

- prove that the hyperbola with equation \( xy = \frac{1}{2}a^2 \) is the hyperbola \( x^2 - y^2 = a^2 \) referred to different axes
- write down the eccentricity, coordinates of foci and vertices, equations of directrices and equations of asymptotes of \( xy = \frac{1}{2}a^2 \)
- sketch the hyperbola \( xy = \frac{1}{2}a^2 \), for values of \( a \), marking on vertices, foci, directrices and asymptotes
- write down the parametric coordinates \((ct, \frac{c}{t})\) for the rectangular hyperbola \( xy = c^2 \), for varying values of \( c \)
- find the equation of the chord joining \( P(cp, \frac{c}{p}) \) to \( Q(cq, \frac{c}{q}) \)
- find the equation of the tangent at \( P(cp, \frac{c}{p}) \)
- find the equation of the normal at \( P(cp, \frac{c}{p}) \)
- find the equation of the chord joining \( P(x_1, y_1) \) to \( Q(x_2, y_2) \)
- find the equation of the chord of contact from \( T(x_0, y_0) \)
- find the point of intersection of tangents and of normals
- prove simple geometrical properties of the rectangular hyperbola including:
  - the area of the triangle bounded by a tangent and the asymptotes is a constant
  - the length of the intercept, cut off a tangent by the asymptotes, equals twice the distance of the point of contact from the intersection of the asymptotes
- find loci of points including:
  - loci determined by intersection points of tangents
– loci determined by intersection points of normals
– loci determined by midpoints of intervals.

**General descriptive properties of conics**
- appreciate that the various conic sections (circle, ellipse, parabola, hyperbola and pairs of intersecting lines) are indeed the curves obtained when a plane intersects a (double) cone
- relate the various ranges of values of the eccentricity \( e \) to the appropriate conic and to understand how the shape of a conic varies as its eccentricity varies
- appreciate that the equations of all conic sections involve only quadratic expressions in \( x \) and \( y \).

**Integration**

**Integration**
- use a table of standard integrals
- change an integrand into an appropriate form by use of algebra
- evaluate integrals using algebraic substitutions
- evaluate simple trigonometric integrals
- evaluate integrals using trigonometric substitutions
- evaluate integrals using integration by parts
- derive and use recurrence relations
- integrate rational functions by completing the square in a quadratic denominator
- integrate rational functions whose denominators have simple linear or quadratic factors.

**Volumes**

**Volumes**
- appreciate that, by dividing a solid into a number of slices or shells, whose volumes can be simply estimated, the volume of the solid is the value of the definite integral obtained as the limit of the corresponding approximating sums
- find the volume of a solid of revolution by summing the volumes of slices with circular cross-sections
- find the volume of a solid of revolution by summing the volumes of slices with annular cross-sections
- find the volume of a solid of revolution by summing the volumes of cylindrical shells
- find the volume of a solid which has parallel cross-sections of similar shapes.

**Mechanics**

**Mathematical Representation of a motion described in physical terms**
- derive the equations of motion of a projectile
- use equations for horizontal and vertical components of velocity and displacement to answer harder problems on projectiles
- write down equations for displacement, velocity and acceleration given that a motion is simple harmonic
- use relevant formulae and graphs to solve harder problems on simple harmonic motion
- use Newton’s laws to obtain equations of motion of a particle in situations other than projectile motion and simple harmonic motion
- describe mathematically the motion of particles in situations other than projectile motion and simple harmonic motion.

**Physical explanations of mathematical descriptions of motion**
- given \( \ddot{x} = f(x) \) and initial conditions derive \( v^2 = g(x) \) and describe the resultant motion
- recognise that a motion is simple harmonic, given an equation for either acceleration, velocity or displacement, and describe the resultant motion.

**Resisted motion**

**Resisted Motion along a horizontal line**
- derive, from Newton’s laws of motion, the equation of motion of a particle moving in a single direction under a resistance proportional to a power of the speed
- derive an expression for velocity as a function of time (where possible)
- derive an expression for velocity as a function of displacement (where possible)
- derive an expression for displacement as a function of time (where possible).
Motion of a particle moving upwards in a resisting medium and under the influence of gravity
• derive, from Newton’s laws of motion, the equation of motion of a particle, moving vertically upwards in a medium, with a resistance \( R \) proportional to the first or second power of its speed
• derive expressions for velocity as a function of time and for velocity as a function of displacement (or vice versa)
• derive an expression for displacement as a function of time
• solve problems by using the expressions derived for acceleration, velocity and displacement.

Motion of a particle falling downwards in a resisting medium and under the influence of gravity
• derive, from Newton’s laws of motion, the equation of motion of a particle falling in a medium with a resistance \( R \) proportional to the first or second power of its speed
• determine the terminal velocity of a falling particle, from its equation of motion
• derive expressions for velocity as a function of time and for velocity as a function of displacement
• derive an expression for displacement as a function of time
• solve problems by using the expressions derived for acceleration, velocity and displacement.

Circular Motion
Motion of a particle around a circle
• define the angular velocity of a point moving about a fixed point
• deduce, from this definition of angular velocity, expressions for angular acceleration of a point about a fixed point
• prove the instantaneous velocity of a particle moving in a circle of radius \( R \), with angular velocity \( \omega \), is \( R \omega \)
• prove that the tangential and normal components of the force acting on a particle moving in a circle of radius \( R \), with angular velocity \( \omega \), need to be \( Mr \omega \) and \( -mR \omega^2 \) respectively.

Motion of a particle moving with uniform angular velocity around a circle
• write down the formulae appropriate for a particle moving around a circle with uniform angular velocity
• apply these formulae to the solution of simple problems.

The Conical Pendulum
• use Newton’s law to analyse the forces acting on the bob of a conical pendulum

• derive results including \( \tan \theta = \frac{v^2}{ag} \) and \( h = \frac{g}{\omega^2} \)
• discuss the behaviour of the pendulum as its features vary
• apply derived formulae to the solution of simple problems.

Motion around a banked circular track
• Use Newton’s laws to analyse the forces acting on a body, represented by a particle, moving at constant speed around a banked circular track

• derive the results \( \tan \theta = \frac{v^2}{Rg} \) and \( h = \frac{v^2d}{Rg} \)
• calculate the optimum speed around a banked track given the construction specifications
• calculate the forces acting on a body, travelling around a banked track, at a speed other than the optimum speed.

Polynomials
Integer roots of polynomials with integer coefficients
• prove that, if a polynomial has integer coefficients and if \( a \) is an integer root, then \( a \) is a divisor of the constant term
• test a given polynomial with integer coefficients for possible integer roots

Multiple Roots
• define a multiple root of a polynomial
• write down the order (multiplicity) of a root
• prove that if \( P(x) = (x - a)^r S(x) \), where \( r > 1 \) and \( S(a) \neq 0 \), then \( P'(x) \) has a root \( a \) of multiplicity \( (r - 1) \)
• solve simple problems involving multiple roots of a polynomial.
Fundamental Theorem of Algebra
• state the fundamental theorem of algebra
• deduce that a polynomial of degree \( n > 0 \), with real or complex coefficients, has exactly \( n \) complex roots, allowing for multiplicities.

Factoring Polynomials
• recognise that a complex polynomial of degree \( n \) can be written as a product of \( n \) complex linear factors
• recognise that a real polynomial of degree \( n \) can be written as a product of real linear and real quadratic factors
• factor a real polynomial into a product of real linear and real quadratic factors
• factor a polynomial into a product of complex linear factors
• write down a polynomial given a set of properties sufficient to define it
• solve polynomial equations over the real and complex numbers.

Roots and Coefficients of a Polynomial Equation
• write down the relationships between the roots and coefficients of polynomial equations of deg 2, 3 and 4.
• use these relationships to form a polynomial equation given its roots
• form an equation, whose roots are a multiple of the roots of a given equation
• form an equation, whose roots are the reciprocals of the roots of a given equation
• form an equation, whose roots differ by a constant from the roots of a given equation
• form an equation, whose roots are the squares of the roots of a given equation.

Partial Fractions
• write \( f(x) = \frac{A(x)}{B(x)} \), where \( \deg A(x) \geq \deg B(x) \), in the form \( f(x) = Q(x) + \frac{R(x)}{B(x)} \), where \( \deg R(x) < \deg B(x) \)
• write \( \frac{R(x)}{B(x)} \), where \( \deg R(x) < \deg B(x) \) and \( B(x) \) is a product of distinct linear factors \( c(x - a_1) \ldots (x - a_n) \), in the form \( \frac{c_1}{x - a_1} + \ldots + \frac{c_n}{x - a_n} \)
• write \( \frac{R(x)}{B(x)} \), where \( \deg R(x) < \deg B(x) \) and \( B(x) \) is a product of distinct linear factors and a simple quadratic factor, in the form \( \frac{c_1}{x - a_1} + \ldots + \frac{c_n}{x - a_n} + \frac{dx + e}{x^2 + bx + c} \)
• write \( \frac{R(x)}{B(x)} \), where \( \deg R(x) < \deg B(x) \) and \( B(x) \) is a product of two different simple quadratic factors of form \( x^2 + b_1 \), in the form \( \frac{c_1x + d_1}{x^2 + b_1} + \frac{c_2x + d_2}{x^2 + b_2} \)
• apply these partial fraction decompositions to the integration of corresponding functions.

Harder Ext 1 Mathematics
Geometry of the Circle
• solve more difficult problems in geometry.

Induction
• carry out proofs by mathematical induction in which \( S(1), S(2) \ldots S(k) \) are assumed to be true in order to prove \( S(k + 1) \) is true
• use mathematical induction to prove results in topics which include geometry, inequalities, sequences and series, calculus and algebra.

Inequalities
• prove simple inequalities by use of the definition of \( a > b \) for real \( a \) and \( b \)
• prove further results involving inequalities by logical use of previously obtained inequalities.
### Term 4 11

<table>
<thead>
<tr>
<th>Test 1</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
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### Term 1 12

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<td></td>
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<td>Trial HSC (Test 9)</td>
<td>Revision including Harder Ext 1 Topics</td>
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*Note: The position of the tests on this timeline, are only suggestive. Actual written notification will be given to every student at least 2 weeks prior to the assessment task.*
# HSC Assessment Grid

## Subject: Mathematics Extension 2

<table>
<thead>
<tr>
<th>Syllabus Components</th>
<th>Syllabus Weighting (%)</th>
<th>Task 1</th>
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<th>Task 3</th>
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### Nature of Assessment Task

<table>
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<th>Written Test (%)</th>
<th>Trial HSC (%)</th>
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Note: The dates of tasks are flexible.

This table should be read in conjunction with the above table and the Extension 2 Timeline.

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(Half Yearly)

(Trial HSC)